APPENDICE

Ideals with a regular sequence as syzygy

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We sketch an alternate approach to Proposition 2, reducing it to results of Huneke and Ulrich [H-U] and Kustin [Ku] (results similar to those of Kustin were also obtained by M. Stillman). In [H-U] the authors work over a ring containing a field, but the results are general, and are done explicitly without this hypothesis in [Ku].

Assume that R is a local Nœtherian ring, that $x_1, \ldots, x_n$ is a regular sequence in R and that $f_1, \ldots, f_n$ are elements of R satisfying the relation

$$(*) \qquad x_1 f_1 + \ldots + x_n f_n = 0.$$

We further set $I = (f_1, \ldots, f_n)$ and suppose that the grade of I is $n-1$, the largest possible value.

If f is a form in $k[x_1, \ldots, x_n]$ defining a nonsingular hypersurface, and if char(k) divides the degree of f, then Euler's relation shows that these hypotheses are satisfied by the partial derivatives of f in the localization of $k[x_1, \ldots, x_n]$.

Theorem. *If grade*$(I) = n-1$, *then*

(i) *if n is odd, R/I is perfect of Cohen-Macaulay type* 2.

(ii) *if n is even, there exists an element $f \notin I$ such that*

$$I : (x_1, \ldots, x_n) = (I, f),$$

and $R/(I,f)$ is perfect of Cohen-Macaulay type 1.

Proof : The most interesting point is the identity of the element f : the relation (*) shows that the vector (f_i) is a linear combination of the syzygies of the x_i. Since the x_i form a regular sequence, their syzygies are given by the first map of the Koszul complex $k : \wedge^2 R^n \longrightarrow \wedge^1 R^n$, so there exists a skew-symmetric matrix A such that $(f_i) = A(x_j)$. The element f is then the Pfaffian of A.

The result follows by specialization from the generic case, which is treated in [H-U], 5.8, 5.9 and 5.12, and in [Ku] . QED

Corollary. *If R is regular, $x_1, \ldots, x_n$ generate the maximal ideal, and g is an element of R such that $ht(I,g) = n$, then the socle of $R/(I,g)$ is two-dimensional.*

Proof : If n is odd, the corollary follows at once from (i). If n is even, it follows from (ii) because g must be a nonzero divisor mod(I, f).

Graded free resolutions for the generic forms of the ideals I and (I, f) as in the Theorem can be found in [Ku], Theorem 6.3. By local duality, this gives the degrees of the socle elements in the corollary (alternatively, one can use linkage, as was done in [H-U]). Applying this to the case of partial derivatives of the equation of a nonsingular hypersurface, one recovers the degree results of Beauville.

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