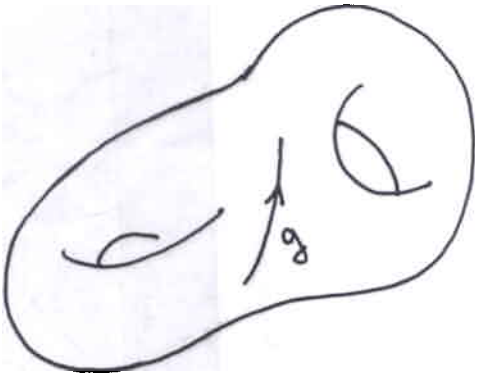
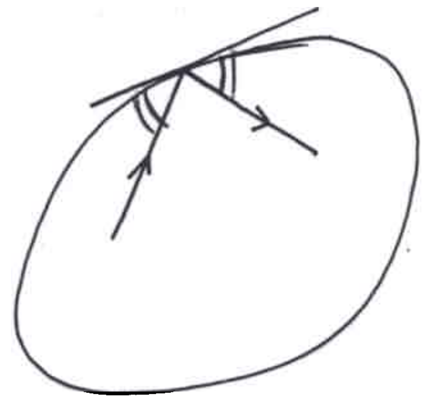


M riemmanian manifold



Q: billiard table.

Classical system :	Quantum system :
- Geodesics (law of reflection)	- Δ self adjoint, positive (boundary condition),
- Periodic geodesics	- Propagator $U(t) = e^{it\sqrt{\Delta}}$

TRACE FORMULA

1. Poisson Relation :

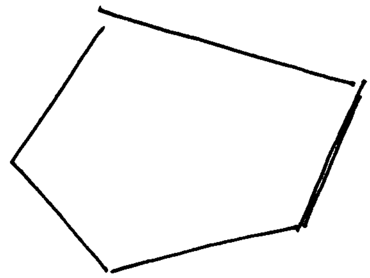
$$\sigma(t) = \mathrm{Tr}(U(t)) \qquad \mathbb{L} = \{lengths\ of\ p.o.\}$$

$$\text{sing. supp}(\sigma) \subset \mathbb{L}$$

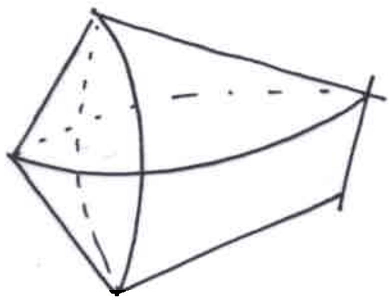
2. Describe (when possible) the asymptotic behavior of :

$$I(s) = \langle \sigma(t), e^{-ist}\rho(t) \rangle,$$

where ρ localizes near $L_0 \in \mathbb{L}$.



Q Euclidean polygon

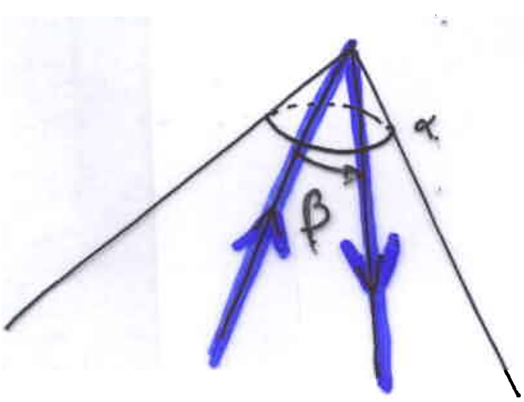


M compact, Euclidean surface with conical singularities

$P = \{conical\ points\}$ and $M_0 = M \backslash P$

Δ is the Friedrichs extension of the Euclidean laplacian on M_0

Diffractive geodesics :



α : angle of the cone,
β : angle of diffraction.

$\mathbb{L} = \{lengths\ of\ (possibly)\ diffractive\ p.o.\}$

theorem 1 *The Poisson relation holds :*

$$\text{sing. supp}(\sigma(t)) \subset \mathbb{L}$$

theorem 2 *The leading order of the contribution of a p.o. in the two following cases, is :*

- *isolated p.o. such that* $\beta_i \neq \pm\pi$

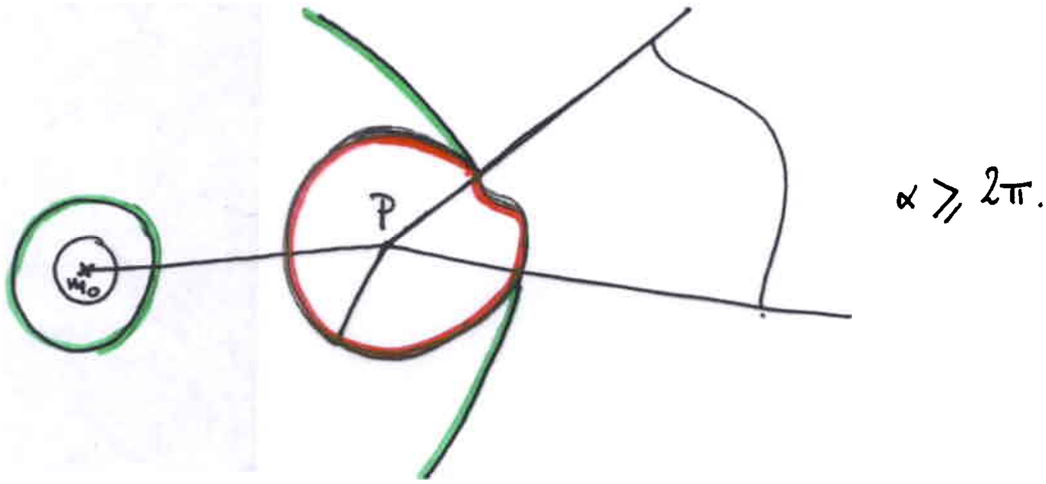
$$I(s) \sim s^{-\frac{n}{2}} c_g L_0 f(L) e^{-isL},$$

$$c_g = (2\pi)^{\frac{n}{2}} e^{-\frac{ni\pi}{4}} \prod \frac{d_{\alpha_i}(\beta_i)}{l_i^{\frac{1}{2}}}$$

- *family of p.o's such that only one diffraction occurs at the boundary*

$$I(s) \sim s^{\frac{1}{2}} \frac{e^{i\frac{\pi}{4}}}{2\pi} \frac{1}{\sqrt{L}} |\mathcal{A}_g| f(L) e^{-isL},$$

Wave equation on an Euclidean cone



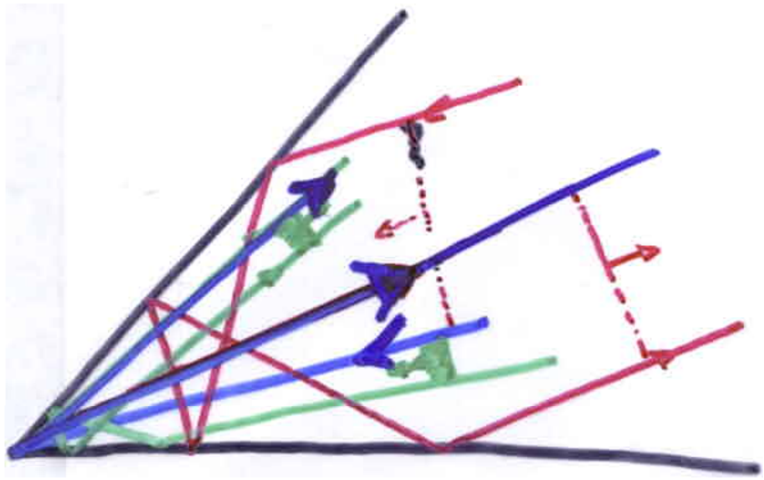
1. Finite speed propagation

2. Apparition of a diffracted front when a singularity hits the vertex

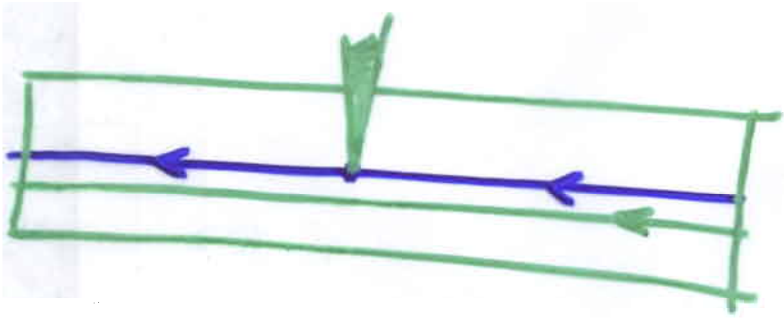
3. A singularity hitting the vertex is instantaneously and integrally reemitted.

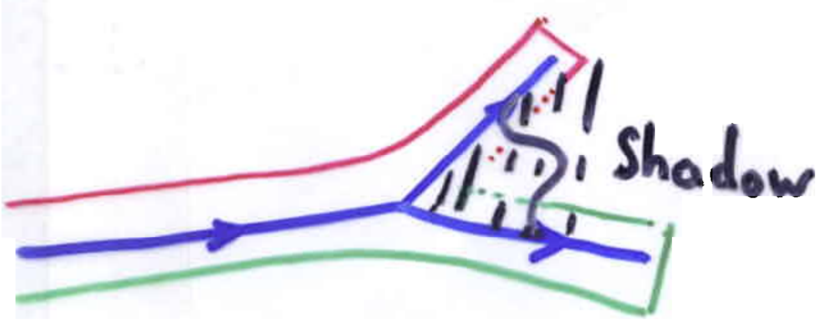
$$WF(u) = WF_0(u) \cup \{p \in P \text{ s.t. } u \text{ not smooth near } p\}$$

One diffraction.



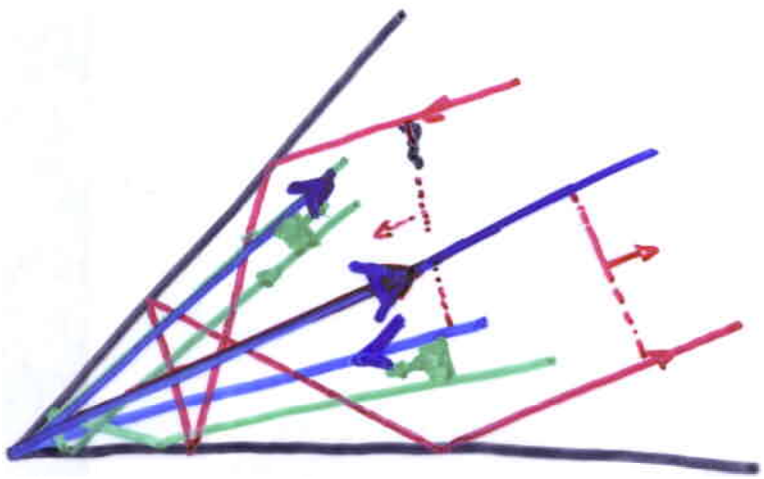
$\alpha \leq 2\pi$



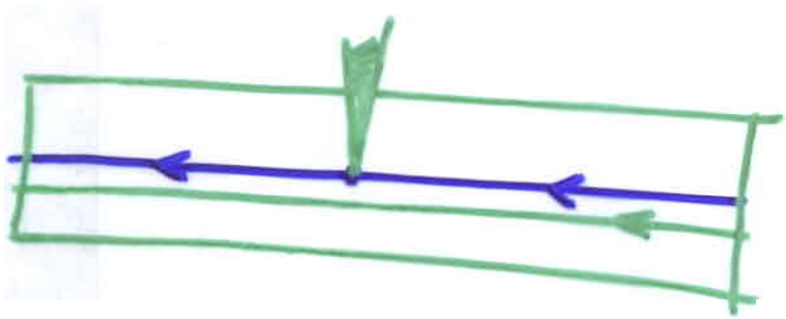


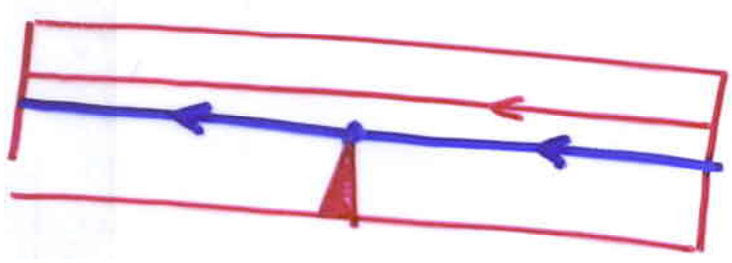
$\alpha \geq 2\pi$

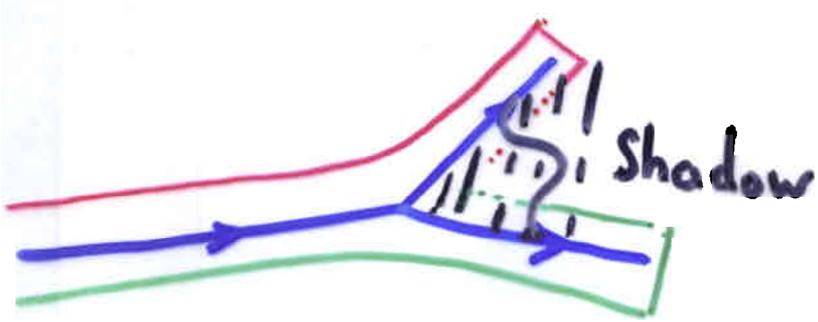
One diffraction.



$\alpha \leq 2\pi$



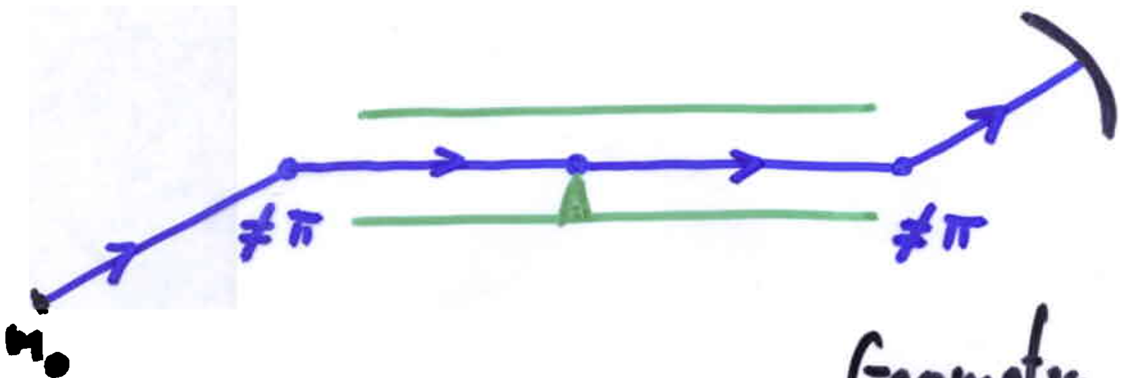




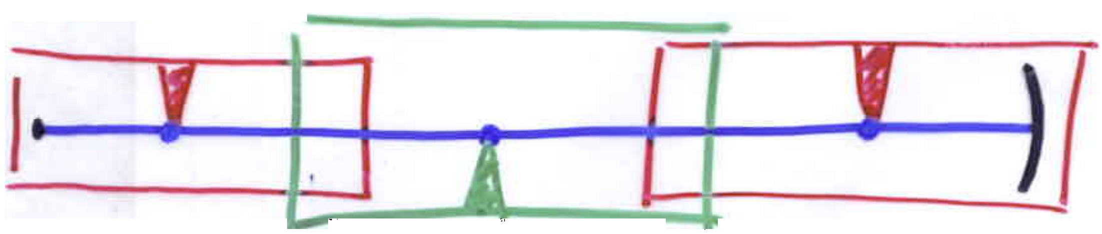
$\alpha \geq 2\pi$

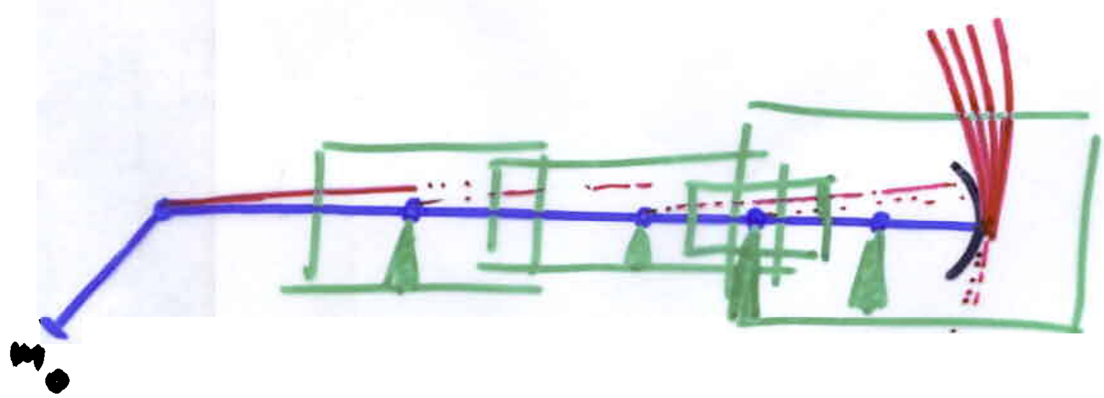
More Diffractions.

Angles of $\neq \pi$ are important, but not only.



Geometry is "good."





Consequences:

- when the geometry is "bad" the propagator surely isn't a FIO.

- Complete description of the periodic orbits:

  - non-diffractive ⇒ interior to a family
  - all angles $+\pi$ (or all angles $-\pi$) ⇒ boundary of a family.
  - any other case ⇒ isolated.

theorem 3 *The singularities of the wave equation propagate along the geodesics.*

The proof relies on the following properties :

- the finite speed of propagation,

- the group law for $U(t)$,

- the definition of geodesics and the behavior near the tip of a cone.

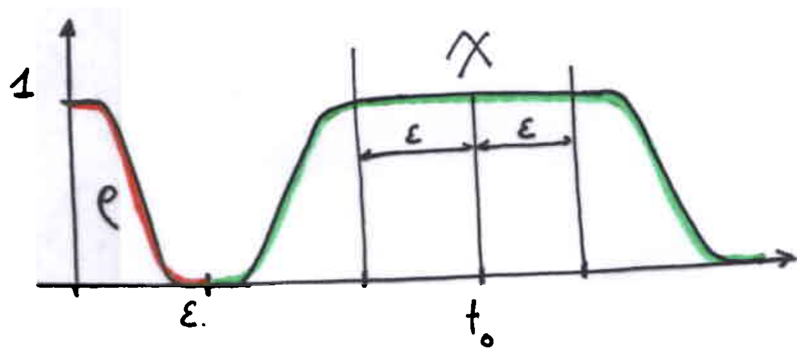
Consequence :
let χ be smooth on M and zero near P, note $\sigma_\chi(t) = \mathrm{Tr}(U(t)\chi)$ then :

$$\text{sing. supp}\sigma_\chi \subset \mathbb{L}.$$

This weak form of the Poisson relation derives from the propagation of singularities by the same wave-front computations as in the smooth case, since all the computations are made above M_0.

To have the Poisson relation, it remains to study the Wave-Front of $\sigma_\rho(t) = \mathrm{Tr}(U(t)\rho)$ where ρ is smooth, is 1 near a conical point and 0 near the others.

Choose t_0, ϵ, χ, and ρ like this :



so that $(1-\chi)U(t_0)\rho$ is smoothing.

$$
\begin{aligned}
\mathrm{Tr}(U(t)\rho) &= \mathrm{Tr}(U(t-2t_0)U(t_0)\rho U(t_0)) \\
&= \mathrm{Tr}(U(t-2t_0)\chi U(t_0)\rho U(t_0)\chi) \\
&\quad + \ smooth \\
&= \mathrm{Tr}(U(t-2t_0)\chi U(2t_0)\chi) \\
&\quad - \mathrm{Tr}(U(t-2t_0)\chi U(t_0)[1-\rho]U(t_0)\chi) \\
&\quad + \ smooth
\end{aligned}
$$

Description of the propagator on the cone : Friedlander's construction.

Look for K_α of the form :

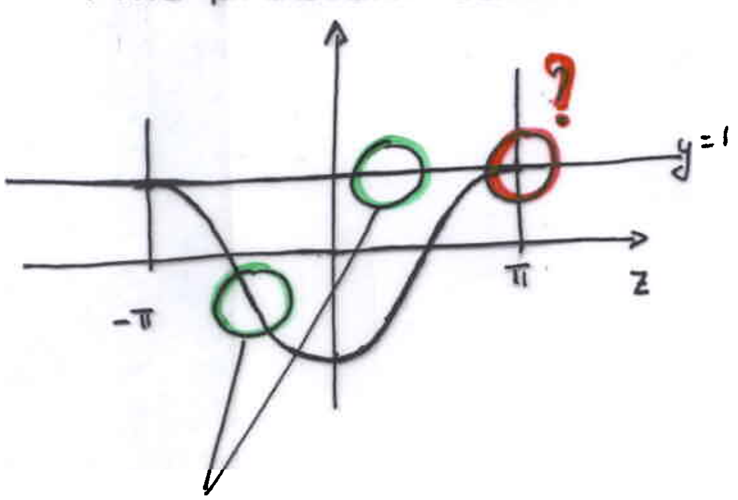
$$K_\alpha(t, R_1, x_1, R_0, x_0) = Per_\alpha \left[\frac{c}{(R_0 R_1)^{\frac{1}{2}}} F^* D_y^{\frac{1}{2}} G(y, z) \right],$$

where G satisfies :

$$G(y, z) = H(y + \cos z) H(|z| \leq \pi) \quad if \quad y < 1$$

$$(1 - y^2)\partial_y^2 G - \partial_z^2 G + y\partial_y G = 0$$

This problem can be solved *explicitely*



G is a lagrangian distribution.