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S_g surface $g \geq 2$

$\Omega \mathcal{M}_g$ = moduli space of abelian differentials (C, ω)

$\mathcal{F}_g$ = isoperiodic foliation.
(Kernel)

(C_0, ω_0) $\{(C_\lambda, \omega_\lambda)\}_\lambda$.

$\int_{a_{\lambda_0}} \omega_{\lambda_0} = \int_{a_\lambda} \omega_\lambda$

$\lambda \sim \lambda_0$
isoperiodic.

$\dim \mathcal{F}_g = 2g - 3$ (of leaves)

e.g. $g = 2$ $C = \{y^2 = x(x-1)(x-x_0)(x-x_1)(x-x_2)\}$

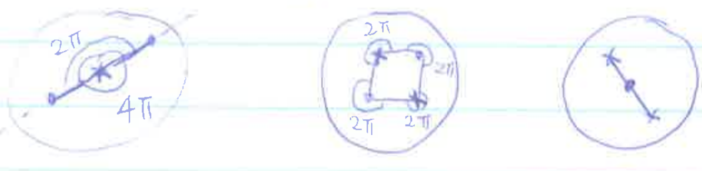
$$\omega = \frac{(ax+b)\,dy}{y}$$

$$\sum \frac{x_i(1-x_i)}{ax_i+b}\frac{\partial}{\partial x_i} + \frac{1}{2}\frac{\partial}{\partial a} - \frac{1}{2}\left(1 + \sum \frac{b(x_i-1)}{ax_i+b}\right)\frac{\partial}{\partial b}.$$

abelian differential $\longleftrightarrow$ branched translations surfaces

(C, ω) $(C, \int \omega)$

Schiffer variations:



McMullen.
Grushevsky
Krichever.

J. W./ Calsamiglia, Francaviglia

Thm: The closure of any leaf ~~of~~ $\mathcal{F}_g$ is

connected component — the set of ~~abelian~~ differentials whose period live in a given closed subgroup $\Lambda \subset \mathbb{C}$ and of a given volume.

① Λ discrete lattice $\rightsquigarrow$ connected component of Hwitz ~~ba~~spaces

② $\mathbb{R} + i\mathbb{Z}$

— Hilbert ~~surface~~ modular surfaces McMullen
(only in genus 2)

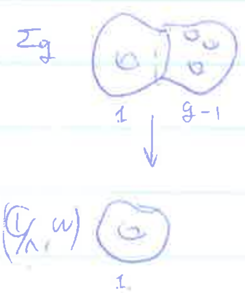
Period map:

$\widehat{\Omega\mathcal{M}_g}$ = moduli spaces of marked abelian differentials

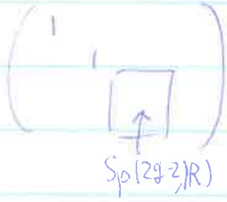
Σ_g: a marking of C is $m: H_1(\Sigma_g, \mathbb{Z}) \xrightarrow{\simeq} H_1(C, \mathbb{Z})$
coming from a diffeomorphism.

$(C, \omega, m) \in \widehat{\Omega\mathcal{M}_g} \longmapsto \int \omega \circ m \in H^1(\Sigma_g, \mathbb{C})$
equivariant w.r.t. modular group $\mathrm{Mod}(\Sigma_g)$

Haupt (1923) $p \in H^1(\Sigma_g, \mathbb{C})$ is in the image of period map
iff (1) $\mathrm{Re}\, P \cdot \mathrm{Im}\, P > 0$. (Rieman's)
(2) p is not a pinching (in homology) to an elliptic abelian differential.



Kapovich (2000) $\qquad Sp(2g,\mathbb{R}) \big/ Sp(2g-2,\mathbb{R})$



Transler principle:

$$\begin{array}{ccccc} & & \hat{\Omega M_g} & & \mathrm{Mod}(\Sigma_g) \circlearrowright \\ & \swarrow^{\pi} & & \searrow^{Per} & \\ (\Omega M_g, \widetilde{F_g}) & & & & H^1(\Sigma_g, \mathbb{C}) \\ \cup & & & & \cup \\ B = \pi(Per^{-1}(A)) & & & & A \quad \mathrm{Mod}(\Sigma_g)\text{-inv.} \\ F_g - & & & & \end{array}$$

This correspondence is bijective iff $Per^{-1}(p)$ are connected for any p.

Thm: $Per^{-1}(p)$ is connected $\forall p \in H^1(\Sigma_g, \mathbb{C})$ $\quad \forall g \geq 2$.

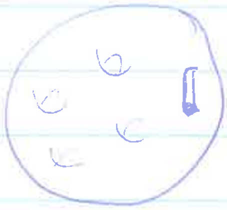
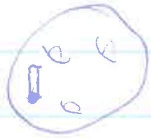
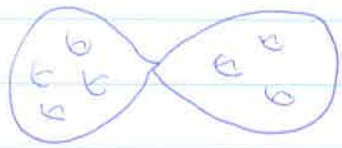
$g = 2, 3$ McMullen (Schottky problem)

$$\overline{Per^{-1}(p)} \underset{\text{bihol}}{\simeq} \mathcal{H}_{g-1} \leftarrow \text{Ziggerspace?}$$

Isoperiodic Degenerations to nodal abelien differentials.

(I)  glue them get a g abelian differential.

$\varepsilon \to 0$  Non injective periods.

[II]   glue them 

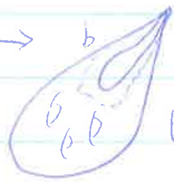
g_1 $\qquad$ g_2 $\qquad$ injective periods.

<u>1st step</u>: Move branch points to a single multiple branch point.

<u>2nd step</u>: Annulus thm of Mazur (1986)

$(C_i, \omega_i, m_i) \qquad i=1,2 \qquad \int \omega_1 \circ m_1 = \int \omega_2 \circ m_2.$

$\exists\, \alpha \in H_1(\Sigma_g, \mathbb{Z}) \setminus \{0\} \qquad \int_\alpha \omega_i = 0$

$\omega_1 \longrightarrow$  ω_1^N $\qquad$  $\leftarrow \omega_2.$ $\quad w_2^N$

<u>Lemma</u>: If (C, ω) and $z \in \mathbb{C}$, the set of homotopy classes of paths $\gamma \in C$ with fixed extrinity and $\int_\gamma \omega = z$ is connected.