Spring 2009

EMISSARY

Mathematical Sciences Research Institute

www.msri.org

Teacher Education Partnerships

Julie Rehmeyer

To train elementary school teachers better, mathematicians and mathematics educators need to work together. This past December, MSRI hosted a two-day workshop on *Using Partnerships to Strengthen Elementary Mathematics Teacher Education*. This was the culminating event in a series of initiatives from a $150,000 grant designed to nurture such collaborations, provided by the S. D. Bechtel, Jr. Foundation. The Carnegie Foundation for the Advancement of Teaching collaborated with MSRI on the project.

Typically, mathematicians and math educators work in almost complete isolation from one another, with the mathematicians teaching the math content classes and the math educators teaching the pedagogy classes. But mathematics and pedagogy are deeply interwoven in teaching practice, and without an integrated curriculum, a student teacher is given little guidance in applying mathematical knowledge to everyday problems in the classroom.

Elementary school teachers need to acquire very different mathematical skills from those in fields like engineering or science. Instead of a large toolset of advanced mathematical techniques,

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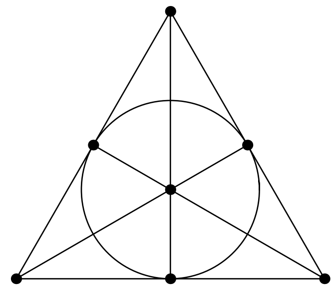
Mathematics educators and research mathematicians learned from each other at the December 2008 MSRI workshop on Teacher Education Partnerships.



Algebraic Geometry at MSRI

Jim Bryan

Algebraic geometry is one of the oldest subjects in mathematics, and yet it is arguably at its most vibrant today. The subject is continually being invigorated by its active connections with topology, complex geometry, representation theory, number theory, commutative algebra, combinatorics, and modern high energy physics. Indeed, the participants of the 2009 MSRI jumbo program in algebraic geometry have a vast range of interests and many of the researchers (including myself) have gravitated to algebraic geometry from other fields.

At the core of algebraic geometry are varieties, spaces defined by polynomial equations. For example, the solution set of a single equation in two variables defines a curve in the plane. If the coefficients of the polynomial are taken to be in the complex numbers, we get a Riemann surface — a one dimensional complex manifold. It was realized early on that rather than studying varieties one at a time, one should consider how they vary in families, or *moduli*.

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Algebraic Geometry

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Riemann studied curves in moduli, and discovered that curves of genus g are parameterized by a $3g-3$ dimensional moduli space. Moduli spaces in general, and the moduli space of curves in particular, now occupy a central place in modern algebraic geometry.

The modern viewpoint in algebraic geometry differs considerably from that of its inception. A significant paradigm shift occurred in the 1960's led by the work of Serre and Grothendieck. They changed the focus from the *points* of a variety to the *functions* on a variety. This brought to the forefront the use of sheaves and homological methods from algebraic topology, and it enlarged the geometric universe from varieties to schemes, which can incorporate arbitrary commutative algebras such as those over number fields.

We are perhaps in the midst of a further paradigm shift. Rather than emphasizing the points of a variety, or the functions on a variety, we can study the *category* represented by the variety. That is we can view each variety X as a moduli space and consider the category $\mathcal{X}$ of families of points in X. This idea gives rise to the notion of algebraic stacks, a further generalization of varieties and schemes. Stacks provide the best language for studying the geometry of moduli spaces, especially those parameterizing objects with non-trivial automorphisms.

Moduli spaces also occur in string theory and in quantum field theory where they often have an algebro-geometric interpretation. In quantum field theory, one purports to integrate an action functional over an infinite dimensional space of fields. In good cases, this integral localizes to an integral over a finite dimensional space of fields: the critical locus for the action functional. For example, the fields in string theory are maps from a Riemann surface to a target space ("world-sheets"). If the target is a Kähler manifold, then the minima of the action function are holomorphic maps and the path integral can be mathematically interpreted as Gromov–Witten theory: integrals over the virtual fundamental class on the moduli space of stable maps. Moduli spaces can also arise in the space of parameters of a string theory or a quantum field theory. For example, string theory predicts that spacetime is 10 dimensional. While four of the dimensions comprise the usual notions of space and time, the remaining six are curled up into Calabi–Yau threefolds — smooth projective complex varieties of dimension three having trivial canonical class. Thus the moduli space of Calabi–Yau threefolds naturally appears within the parameter space of string theory.

Some of the most exciting recent advances in the theory of moduli have been fueled by the interactions between high energy physics and algebraic geometry. Inspired by physicist Michael Douglas's notion of Π-stability, Tom Bridgeland defined the space of stability conditions on the derived category of coherent sheaves on a variety. The derived category enlarges the category of sheaves on a variety to include complexes of sheaves. Originally conceived as an ultra-efficient language to handle techniques from homological algebra, it has recently become the focus of study in its own right. Bridgeland stability generalizes the notion of slope stability for sheaves to objects in the derived category. The set of Bridgeland stability conditions forms a parameter space which is a mathematical model for the string theory notion of "complexified Kähler moduli space".

The study of derived categories and Bridgeland stability conditions has recently exploded behind the exciting work of Joyce, Kontsevich and Soibelman, and others. They have begun the program of "counting" Bridgeland stable objects in the derived category of coherent sheaves on a Calabi–Yau threefold. These counting invariants are generalizations of the holomorphic Chern–Simons invariants defined by Donaldson and Thomas in the late nineties. Donaldson–Thomas invariants were famously conjectured to be equivalent to Gromov–Witten invariants by Maulik, Nekrasov, Okounkov, and Pandharipande in 2003. Now, using the framework developed by Joyce, Kontsevich and Soibelman, one can study the structure of Donaldson–Thomas invariants by using wall-crossing formulas to determine how the invariants change as the stability condition varies. This turns out to be very powerful both computationally and conceptually. In their talks in the "modern moduli theory" workshop, Toda and Bridgeland each employed these ideas to prove well known conjectures in the subject.

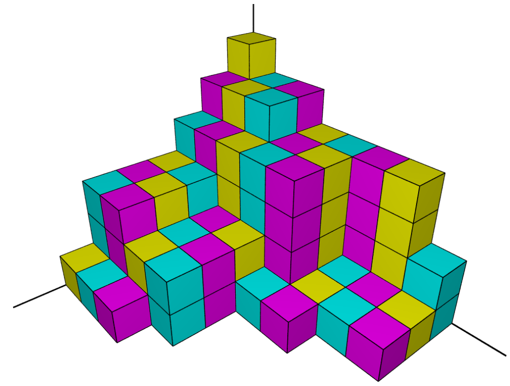
Remarkably, the sophisticated Donaldson–Thomas counting invariants can sometimes be computed by concrete, combinatorial means. If the moduli space of sheaves admits a torus action with isolated fixed points, the work of Behrend and Fantechi shows that the associated Donaldson–Thomas invariant is simply given by a (signed) count of the fixed points, which can often be described combinatorially. For a prototypical example, consider the simplest of all Calabi–Yau threefolds: $\mathbb{C}^3$. The moduli space in question is the Hilbert scheme of n points in $\mathbb{C}^3$. It parameterizes ideals in the ring of functions on $\mathbb{C}^3$ whose quotient has dimension n:

$$\mathrm{Hilb}^n(\mathbb{C}^3) = \{I \subset \mathbb{C}[x,y,z] : \dim \mathbb{C}[x,y,z]/I = n\}.$$

The action of the complex torus $(\mathbb{C}^*)^3$ on $\mathbb{C}^3$ induces an action on $\mathrm{Hilb}^n(\mathbb{C}^3)$ which has isolated fixed points. Indeed, it is easy to see that the only torus fixed ideals are those generated by monomials in x, y, and z. In turn, monomial ideals are in bijective correspondence with *3D partitions*, piles of boxes stacked stably in the corner of a room. If we think of location of the boxes as labelled by tuples (i,j,k) of non-negative integers, then the 3D partition π corresponding to a monomial ideal is given as follows:

$$\pi = \{(i,j,k) \in \mathbb{Z}^3_{\geq 0} : x^i y^j z^k \notin I\}.$$

A sample 3D partition looks like this:



The n-th Donaldson–Thomas invariant of $\mathbb{C}^3$ is thus given by a signed count of 3D partitions of size n. The sign turns out to be simply given by the parity of n

$$\mathrm{DT}_n(\mathbb{C}^3) = (-1)^n \,\#\{\text{3D partitions of size } n\}.$$

In 1916, Percy MacMahon found a formula for the generating function of 3D partitions. Applying his result, one obtains

$$\sum_{n=0}^{\infty} \mathrm{DT}_n(\mathbb{C}^3)\, q^n = \prod_{m=1}^{\infty} \left(\frac{1}{1-(-q)^m} \right)^m .$$

The above explicit formula for the Donaldson–Thomas partition function of $\mathbb{C}^3$ generalizes in several interesting ways. Replacing $\mathbb{C}^3$ by an arbitrary toric Calabi–Yau threefold, one is led to the *topological vertex*. It is a formalism for computing the Donaldson–Thomas partition function of toric Calabi–Yau threefolds. Its central object is the vertex, the generating function which counts 3D partitions which are allowed to have boxes extending to infinity along the coordinate axes. By considering *orbifold* toric threefolds, one is led to counting 3D partitions whose boxes are colored by representations of a finite group. For example, the boxes in the figure are colored by the characters of $\mathbb{Z}_3$ and it corresponds to a torus fixed point in the Hilbert scheme of the orbifold $\mathbb{C}^2/\mathbb{Z}_3 \times \mathbb{C}$. Alternatively, $\mathrm{Hilb}^n(\mathbb{C}^3)$ can be viewed as a moduli space of quiver representations, and generalizing this example to other quivers leads to more exotic combinatorial objects such as *pyramid partitions*. The associated Calabi–Yau geometries are *non-commutative* threefolds.

Of course our discussion of moduli spaces, physics, stability conditions, and Donaldson–Thomas theory represents only a fraction of the recent progress in modern algebraic geometry. Other hot topics include the minimal model program, tropical geometry, and log geometry. While this article is necessarily biased by the author's tastes and interests, its subject is representative of some general features of algebraic geometry — it is a mixture of the classical and the modern, and is continually finding surprising new connections to other parts of mathematics.

The MSRI Mathematical Circles Library

James Sotiros

In November 2006, MSRI led a NSF-sponsored observational group to St. Petersburg and Moscow to study Math Circles. Eastern Europe has a 100 years history with Math Circles that have helped make it one of the richest areas for mathematical genius in the world. We went to learn from their experience and translate their programs to help solve the United States' difficult problems in identifying and mentoring mathematical talent in young people.

Math Circles cultivate interest and aptitude in math by bringing together mathematicians (University faculty, undergraduates, graduates, postdocs, retirees) with pre-college students (and sometimes their teachers) for a rich, lively and engaging introduction to mathematics. Math Circles happen after school and are extracurricular, problem-based enrichment programs for kids who love math.

In Russia, the trip's activities centered around the fabled MCCME (Moscow Center for Continuous Mathematical Education) with talks and visits to several sites that help make up the considerable structure of gifted mathematical education in Russia including math circles, Olympiads and other contests, math camps and special math schools, including the storied Moscow School 57.

A remarkable wealth of literature has been created to support the Russian circles, Olympiads, summer math camps and special schools. The Russian texts include brilliantly crafted problems, fun and easily accessible on the surface but full of mathematical adventure and learning opportunities for those who want to and can dig deeper.

MSRI has joined with the American Mathematical Society and the John Templeton Foundation to translate into English, edit, publish, and market some of the best of these Russian books. These books, along with two American books on circles, will start a new AMS book series called the MSRI Mathematical Circles Library.

The MCL and its activities are guided by a special advisory board, chaired by Tatiana Shubin of San Jose State University, composed of many distinguished mathematicians, scientists and educators:

- Zuming Feng, Phillips Exeter Academy
- Tony Gardiner, University of Birmingham, England
- Kiran Kedlaya, Massachusetts Institute of Technology
- Nikolaj N. Konstantinov, Cofounder of the Independent University of Moscow (IUM), Chair of the Coordinating Council of IUM, MCCME Board of Trustees
- Silvio Levy, MSRI Book Series Editor (and MSRI Librarian emeritus); Editor for MSP (Mathematical Sciences Publishers)
- Walter Mientka, First Director of the American Math Competitions (AMC)
- Bjorn Poonen, Massachusetts Institute of Technology
- Alexander Shen, Directeur de recherche, CNRS, Marseille LIF, Senior researcher, Moscow Institute of Information Transmission Problems
- Tatiana Shubin, San Jose State University; Director, San Jose Math Circle and Bay Area Mathematical Adventures
- Zvezdelina Stankova, Mills College, Director, Berkeley Math Circle
- Ravi Vakil, Stanford University, Stanford Math Circle Faculty Coordinator
- Ivan Yashchenko, Director of the MCCME, Vice-Rector of the Moscow Institute for Improving Teachers' Qualification, Vice-President of the Organizing Committee of the Moscow Math Olympiad
- Paul Zeitz, University of San Francisco; Director, San Francisco Math Circle
- Joshua Zucker, MSRI, Director of the Julia Robinson Mathematics Festival

Here are some of the books selected for translation:

Moscow Mathematical Olympiads, 1993-2005, edited by Fedorov, Kanel-Belov, Kovaldzhi, and Yashchenko: One of a new generation of problem books, offering not only problems and solutions, but hints, extensions, and suggestions for work with students.

Children and Mathematics, by Zvonkin: A remarkable account of the experiences of one Russian mathematician with children of school and pre-school age. Written as an account of one person's experiences, it relates this experience to deeper issues within mathematics and to related literature in the field of cognitive psychology.

Problems in Plane Geometry and Problems in Solid Geometry, by Prasolov: Two encyclopedic sets of problems by one of Russia's masters of the form. Sorted by mathematical topic, with solutions and occasional hints.

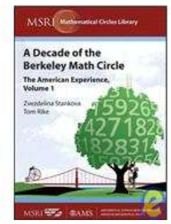
Invitation to a Math Festival, by Yaschenko: Challenging problems for younger students.

Moscow Math Circle Curriculum in Day-by-Day Sets of Problems, by Dorichenko: This book is uniquely suitable for people who are just starting a circle because the sets are very well balanced and checked in real circles; the material is coherent and represents a continuous development of several topics such as geometry, combinatorics, algebra, and number theory throughout two years of circle meetings.

Lessons in Elementary Geometry, a Teacher's Companion, by Hadamard: This is the companion to the recent English translation (by Mark Saul, and also published by the AMS) of a classic text by one of the great mathematicians of the twentieth century.

Two books published in this series were originally written in English, and have already appeared:

Circle in a Box, by Sam Vendervelde. A remarkable how-to publication for starting a math circle, complete with ideas for location, recruitment of instructors and students, funding, and relations with parents.



A Decade of the Berkeley Math Circle: the American Experience, vol. 1, edited by Zvezdelina Stankova and Tom Rike. Selected sessions from ten years of the pioneering Berkeley Math Circle, contributed by a number of mathematicians. Further volumes are in the works.

These books will help support mathematicians at every level to involve themselves with math circles. In doing so they will engage with often yet unidentified mathematically talented youngsters that will bring continued vitality and success to the field of mathematics and to our nation's scientific endeavors.

We welcome suggestions of books to be considered for future publication. Furthermore, we encourage our readers to get involved as potential translators and/or editors. Please contact Tatiana Shubin at shubin@math.sjsu.edu,

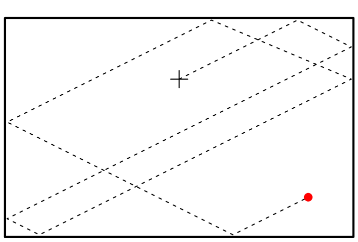
The Fall 2008 Programs

The *Emissary* appeared only once in 2008, and as a consequence there was no opportunity to cover the very rich and exciting Fall 2008 programs. The editor hopes this writeup, based largely on the final reports of the program organizers, will give the reader an idea of the diversity and impact of last fall's research and academic activities. See the front page for this spring's program.

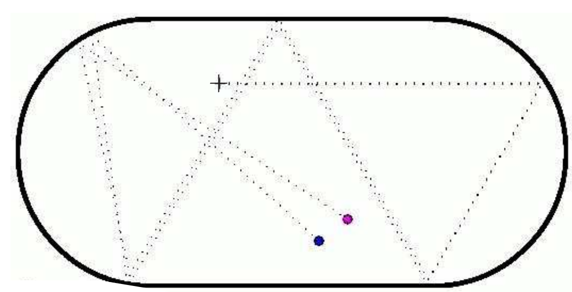
Ergodic Theory and Additive Combinatorics

Ergodic theory deals with systems whose evolution is "well-mixed", in the sense that there are no portions of the space of states that remain isolated from the rest if we allow the system to evolve long enough.

For example, the trajectory of a billiard ball in a rectangular table is not ergodic, because only a few directions of motion result from any given initial state (position and direction).

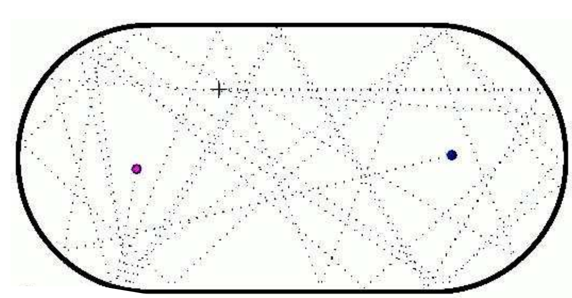


But in a stadium-shaped billiards table, the rounded ends introduce mixing, and a single trajectory will fill up the space of states. And note how two trajectories starting due right from two points barely apart (both under the cross) diverge after 5 seconds…



http://tinyurl.com/28p7xp

and are completely estranged after a few more.

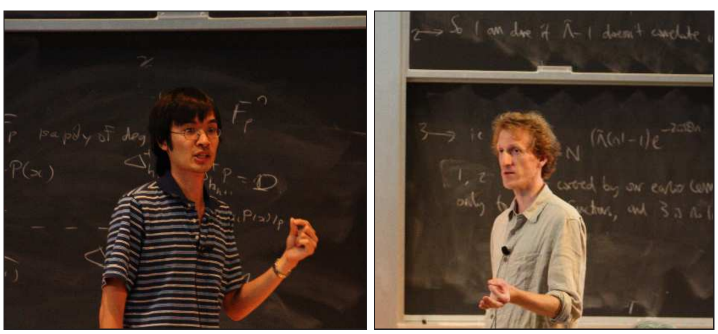


David M. Harrison

Much recent work in ergodic theory has been motivated by interactions with combinatorics and with number theory. For example, Szemerédi's Theorem states that a set of integers with positive upper density contains arbitrarily long arithmetic progressions. The original argument was an intricate use of combinatorics; a second proof was given by Furstenberg using ergodic theory and more recently, Gowers gave a third proof based on Fourier analysis. In the last few years, methods of combinatorics, number theory, harmonic analysis, and ergodic theory have been combined to attack old problems on patterns, such as arithmetic progressions, in the prime numbers. Likewise, a recent result by Ben Green and Terry Tao on arbitrarily long arithmetic progressions in the set of primes immediately attracted the attention of ergodic theorists.

To accommodate this wealth of "interdisciplinarity", the program on Ergodic Theory and Additive Combinatorics, organized by Ben Green (University of Cambridge), Bryna Kra (Northwestern University), Emmanuel Lesigne (University of Tours), Anthony Quas (University of Victoria), and Mate Wierdl (University of Memphis) brought together, besides the organizers, 11 postdocs, eight graduate students in residence, and dozens of workshop participants.

Two introductory workshops opened the program: the now-traditional "Connections" workshop set the stage and was aimed particularly at graduate students and postdocs in harmonic analysis, combinatorics and ergodic theory, while the week-long "Introduction to Ergodic Theory and Additive Combinatorics" included minicourses by Bernard Host, Ben Green and Terry Tao, who each walked listeners from carefully explained basic facts to recent results and sketches of their proofs.



Emmanuel Roy

Terry Tao and Ben Green were two of the minicourse lecturers.

The third workshop, according to the organizers' final report, "was a high level conference on rigidity theory, which lies at the intersection of several mathematical fields. . . . The term 'discrete rigidity phenomena' was invented specially for this workshop . . . Although many of the invited speakers were bemused (or occasionally amused) by the title, they all gave talks very much within the intended spirit of the workshop. This strongly suggests that the time was ripe for such a meeting."

As usual, the atmosphere was one of excitement and intellectual give-and-take. "There were numerous informal and lively discussions, varied and interesting questions circulated (both in formal problem sessions and informal exchanges), and new collaborations began. This atmosphere of scientific exchange was confirmed by numerous comments by participants. The general organization of MSRI, including the excellent library, make the Institute a great place for dynamical mathematical research," wrote the organizers.

What came out of it?

A notable feature of the [Ergodic Theory and Additive Combinatorics] program was the large number of questions in circulation. Amongst those posing questions, Michael Boshernitzan stands out for having a steady supply of innocent-sounding questions exploring the limits of the theory.

While one or two of these were answered during the program (e.g. the paper of Boshernitzan and Glasner), the majority were taken home by participants where they will no doubt continue to plague people. One question formulated in a particularly elementary way sounded so innocent that on the day after Thanksgiving (after which a number of the participants were due to leave), there was a veritable maelstrom of activity with several members unsuccessfully proposing methods of attack. The original question, alas, escaped to torment members another day (although it seems now that there is a solution to this problem).

The central idea in the recent developments (in the last 10 years) of the subject is that of Gowers norms or equivalently on the ergodic side, the Host-Kra seminorms. At a heuristic level, these leads to a decomposition of sets and functions into structured and "random" parts. An emerging idea in recent years has been the so-called inverse conjecture for the Gowers norms, where one is seeking to express in a quantitative way in terms of correlations what it means to be have large Gowers norm. During the program, a major project of Bergelson, Tao and Ziegler was completed establishing the inverse Gowers conjecture in the case of $\mathbb{F}_p^d$. Their result may be informally stated as follows: *If f has biased kth derivative then f correlates with a polynomial phase of degree $k-1$.* This leaves open the major question of the inverse conjecture of Green and Tao for $\mathbb{Z}/N\mathbb{Z}$.

– From the organizers' final report

Analysis on Singular Spaces

Singularities appear in many fields of mathematics, of course with different properties in each. For example, singular varieties in algebraic geometry not only occur naturally as fundamental objects themselves, but even the moduli spaces of smooth varieties are naturally singular. Seemingly smooth, noncompact objects often become singular spaces upon compactification: Euclidean space can be radially compactified to a manifold with boundary, the simplest possible "singular space," while the configuration space for k-particle dynamics on $\mathbb{R}^n$ naturally has a compactification as a kn-dimensional manifold with corners. Smooth symmetric spaces often have natural compactifications, such as the Borel–Serre compactification, that are manifolds with corners. And objects with irregular boundaries occur frequently in mathematical physics: classical problems like the scattering of waves by a slit already involve singular geometries. Singular structures are moreover thought to play an important role in the scattering of seismic waves through the interior of the earth; the associated inverse problem is of manifest practical importance.

It turns out that many analytic constructions and a variety of results on differential equations can be extended from the setting of smooth manifolds to singular spaces of various sorts. Many of these generalizations bring forth important connections with other fields: for example, the study of elliptic equations on singular spaces has had fruitful interaction with topology, while the subject of spectral and scattering theory on singular spaces spans areas as diverse as number theory (modular forms) and physics (many-body scattering, relativity).

Many areas of analysis on singular spaces have in common the use of asymptotic expansions of solutions to partial differential equations near singular strata. Tools developed by different teams and subspecialties sometimes turn out to be based on the same idea in different guises. MSRI's Fall 2008 program devoted to Analysis on Singular Spaces aimed to bring together researchers in these diverse fields and to facilitate the sharing of mathematical techniques, with the ultimate goal of fostering a systematic and general theory of partial differential equations on stratified spaces, using iterative techniques to peel away successive strata.

The program was organized by Gilles Carron (University of Nantes), Eugénie Hunsicker (Loughborough University), Richard Melrose (Massachusetts Institute of Technology), Michael Taylor (University of North Carolina, Chapel Hill), András Vasy (Stanford University) and Jared Wunsch (Northwestern University). Here are some of the breakthroughs achieved during the semester, as presented in the organizers' final report:

"Tanya Christiansen and Michael Taylor proved a new result on inverse-scattering for obstacles in waveguides, following on a talk that Christiansen gave on some results in this direction. The inverse-scattering problem is that of determining an object — in this case, one in the middle of a waveguide — by bouncing waves off of it; these waves might be acoustic, seismic, or electromagnetic: to a good approximation, the theory is the same. The work of Christiansen-Taylor allows us to determine the shape of the obstacle, subject to some technical hypotheses, by using waves of a small range of wavelengths. Previous results of Christiansen had been confined to the two-dimensional case.

Frédéric Rochon reported that a casual conversation with Daniel Grieser at the beginning of the semester later led to decisive progress in his project with Pierre Albin on the index of $\bar{\partial}$ operators acting on stable parabolic vector bundles of degree zero. Index theory is a subject of crucial importance both in geometry and in modern mathematical physics, where it arises in connection with the study of anomalies in quantum field theory.

Andrew Hassell discussed and, in interaction with Luc Hillairet, was able to extend his recent breakthroughs on the failure of quantum unique ergodicity for the Bunimovich stadium. These results show that while the motion of a billiard ball in the stadium is rather chaotic, nonetheless there can be quantum states at high energy that are narrowly concentrated along those billiard trajectories that bounce back and forth within the rectangular part. The existence of these quantum states had been a major open problem in the burgeoning field of quantum chaos."

What's a singular space?

Our program had lots of visitors with different answers to this question, but roughly speaking, anything that's not a smooth boundaryless manifold ought to qualify. Manifolds with boundaries can be viewed as the beginning, and from the point of view of PDE they offer plenty of scope and interesting phenomenology, from the Atiyah-Patodi-Singer index theorem in the world of elliptic equations, to the challenges of wave propagation in the hyperbolic setting. More generally, some form of stratification structure is common to many essential examples of singular spaces (this is easy to see in a manifold with corners, where each boundary face is in turn a manifold with corners).

One extreme of scientific activity was on a space with plenty of structure, the Bunimovich stadium, which only barely fails to be a smooth manifold with boundary. Andrew Hassell recently made a breakthrough showing that almost every Bunimovich stadium fails to be quantum unique ergodic, dispatching a longstanding open problem in quantum chaos. Considerable research activity attended this result and its consequences. A much more singular setting is that considered by Albin, Leichtnam, Mazzeo, and Piazza in their ongoing work on signature theorems: they focus on a very general class of stratified spaces satisfying a certain topological condition (the Witt condition). At the far extreme, we had some bona fide singularity theorists including Terence Gaffney and David Trotman to remind the analysts of how much on heaven and earth is (so far) undreamt of by the PDE community.

– Jared Wunsch, Program organizer

MSRI's Simons Auditorium: in many ways a singular space

This stadium billiard is the same one mentioned back on page 4. From this you can see that there was a nontrivial intersection between the two fall programs. There was also considerable interaction with the Topology of Stratified Spaces Workshop held at MSRI in September; while this workshop was not formally a part of either program's activities, it brought in many experts in the more topological aspects of singular spaces, and went a large distance to realizing the goal of fostering interaction between the analysis and topology communities.

Teacher Education Partnerships

(continued from page 1)

teachers need a deep, flexible, and intuitive understanding of basic mathematics, and they must be able to translate that understanding into words and images that a young child can understand.

A mathematician who hasn't thought deeply about elementary school pedagogy would usually have little idea how to give future teachers the mathematical skills they need in practice. But mathematicians are essential to the training of teachers because elementary school students need to learn not just how to perform mathematical algorithms but also how to think mathematically, and they need to be exposed to the beauty and delight of mathematics.

Unfortunately, partnerships between mathematicians and math educators face significant barriers. Experts in the two fields have few incentives to work together. Developing a collaboration requires a sustained effort over many years, which may take mathematicians away from teaching advanced courses and from their core research. The administrative obstacles can be formidable.

Nevertheless, fruitful collaborations have sprung up at several campuses across the country, and as a result of this grant, more have been formed. Furthermore, the grant provided an opportunity to study the experiences of the pioneers to help to make it practical for everyday mathematicians and mathematics educators to form collaborations, not just a few extraordinarily dedicated ones. Here are some of the experiences of these pioneering collaborations.

University of Nebraska-Lincoln

Every semester, Jim Lewis used to read student evaluations for all mathematics courses taught at the University of Nebraska-Lincoln as part of his job as department chair. For years, he saw the same thing: Instructors in the mathematics courses for elementary school teachers received terrible evaluations. "This course is irrelevant for my work as a teacher," the student teachers would say. "Why do I have to learn this?"

At the same time, he heard from his instructors about how weak the student teachers' mathematical preparation was, even about essential concepts like place value.

So when Lewis decided to teach the math classes for elementary school teachers himself, he'd have to figure out how to connect the mathematics to teachers' everyday work in the classroom — without having ever taught elementary school himself. He also knew from the evaluations that the student teachers didn't have a lot of respect for mathematicians.

In 1999, he asked Ruth Heaton, a young professor in UNL's Department of Teaching, Learning, and Teacher Education, to collaborate with him. She could help him understand the needs of student teachers while convincing them that the mathematical work was important. Heaton was thrilled to collaborate, since she'd been seeing the problems in their mathematical preparation as well.

Most of the student teachers, Heaton and Lewis found, had a rigid, algorithmic understanding of mathematics. They usually knew only one way to solve a problem and couldn't imagine that there might be multiple ways of doing it. Many were afraid of mathematics. Others figured teaching mathematics would be a breeze — after all, they said, mathematics is just a matter of following rules.

The pair realized they needed to radically change their students' conception of mathematics. They wanted their new teachers to have rich mathematical habits of mind: to understand which tools are appropriate when solving a particular problem, to be flexible in their thinking, to use precise mathematical definitions, to be able to explain their solutions to others, and to be persistent.

Accomplishing this, they realized, would require more than a course or two. They needed an "immersion semester" when their students were entirely focused on the teaching and learning of mathematics. They designed a block of four courses, for a total of ten hours of classes, all with an emphasis on mathematics teaching and learning: a mathematics content class, a mathematics pedagogy class, a field experience that involved working in an elementary classroom two days each week, and a class with master teachers. The pair worked to integrate the classes, creating a common syllabus and, when possible, common assignments.

When Lewis and Heaton first started their program as a pilot project, they found that the students who went through it went on to do better in the rest of their classes and in their student teaching as well. They are now working to expand their efforts to include middle-school and practicing teachers.

Shannon Parry

Jim Lewis and Ruth Heaton

University of Michigan

At the University of Michigan, educator Deborah Loewenberg Ball and mathematician Hyman Bass collaborate both in teaching future teachers and in their research. Learning to teach mathematics to young children, they argue, demands not only knowledge but also skills that take practice to acquire, just as in gymnastics or surgery or music. Sure, mathematics teachers need to know mathematics, but even more, they need to know how to use their mathematical knowledge in teaching. Ball and Bass are building both a theory and a set of practical tools to support teaching practice.

Ball began work to identify the mathematical demands on teachers by studying teachers at work. She assembled lots of records of real-life teaching, including videos of every lesson in an entire year of third-grade instruction. Shortly after Ball and Bass met, she recruited him to go through those records to identify all of the mathematically significant events, figuring a mathematician might well see different things in the videos than an educator would.

Deborah Ball teaching elementary school children.

Indeed he did. For example, Bass noticed the enormous care teachers need to put into the use of mathematical language. Informal language helps make the mathematics more accessible, yet mathematics relies on precise use of terms. The task of balancing these demands is made more difficult because even curriculum materials can use vague language that is ambiguous or incorrect. An even number, for example, might be explained as one that can be divided into two equal parts. But this definition is too loose, since the number 7 can be divided into $3\frac{1}{2}$ and $3\frac{1}{2}$. Children can get confused by this kind of vague language.

Once Bass, Ball and their collaborators had developed their theory about the mathematical knowledge teachers need, they created tests to evaluate it. These tests helped to validate their theory: if a teacher scored well on the tests, their students did indeed tend to have higher levels of achievement.

Ball and her team are now designing teacher education curricula based on their findings. They have also developed an array of materials that all sections of their class use, including slides, in-class tasks, questions, assignments, and tests, which they plan to make available to groups at other universities.

Sonoma State University

Unlike the other schools in the project, Sonoma State has long had both mathematicians and mathematics educators within the mathematics department. Such collaborations are far more common at state universities with large teacher education programs than at research universities, the Sonoma State professors say. Since the majority of elementary school teachers are trained at state universities, these collaborations are particularly influential for the elementary school teaching profession.

Still, the Sonoma State professors used the grant to make their collaboration deeper. Typically, the instructors of their mathematics content course have a common syllabus and common expectations and stay in close contact informally over the course of the semester. They have not, however, closely coordinated how they've taught each section.

A team of five professors in the mathematics and education departments (Rick Marks, Edith Prentice Mendez, Kathy Morris, Ben Ford and Brigitte Lahme) used the grant as an opportunity to embark on an intensive "lesson study," closely scrutinizing their instruction. They planned their classes jointly in great detail, attended one another's classes, and met between each section of the class to discuss it and adjust their plan.

The team focused its efforts on a two-and-a-half-week unit they had used successfully for many years to get students to understand place value and base ten numbering systems through inventing the base five system for themselves. They told a story about a prehistoric tribe that counted using the letters A to Z, which corresponded to 1 through 26. Beyond 26, they just said "many." The tribe had begun to need to count higher, and the students were assigned the task of inventing a new system that used just the symbols A through D and a new one, 0.

The common difficulty was that students would devise variations on a Roman numeral system rather than a base five system. The team devised various strategies to nudge the students away from the Roman numerals, including using manipulatives (a block, a rod of five blocks, and a flat of 25 blocks) and giving additional clues (like that A still meant 1 and B still meant 2). They also realized from past experience that students who did manage to develop a base 5 system for themselves, even with extensive hints, did better for the rest of the semester. So while in the past they'd been content if at least one group came up with base 5, which was then adopted by the whole class, they made it their goal for all the groups in the class to work it out. They became far more directive than they had in the past — with better results.

The instructors were surprised to end up making such substantial changes to a unit they'd done with significant success for years. They also used the lesson study as an opportunity to examine their process of collaboration.

Mills College

As a result of the grant from the S. D. Bechtel, Jr. Foundation, mathematics educator Ruth Cossey of Mills College and Barbara Li Santi, a mathematician at the same institution, cotaught a math course for future elementary school teachers for the first time.

Their students, they found, had met with a lot of damage to their mathematical identities. So the pair emphasized equity, in the spirit of the civil rights movement. They had the students write mathematical autobiographies. The students discussed the kind of learning environment they preferred and things they particularly did or didn't want to hear while they were doing mathematics. This led to an agreed-upon set of norms for the classroom, which Cossey and Li Santi light-heartedly enforced during the semester.

The course included work on traditional concepts like place value, but it also included basic logic, since the pair found that their students had a hard time following a mathematical argument. One way they did this was through cooperative logic puzzles, where the students were given clues to a puzzle they solved as a group. Then they challenged the students to explain their reasoning.

The translation of mathematics to and from language was a key component of the course. When students offered an answer in class, Cossey and Li Santi would ask "Are you sure? Why?", regardless of whether the answer was right or wrong.

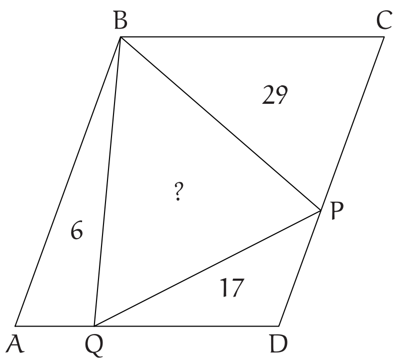
Since then, Cossey has continued to teach the course frequently using the methods the pair developed together, and Li Santi was able to join in part of the course one other semester. Regular coteaching, however, has proved difficult to arrange.

Partnerships between mathematicians and mathematics educators are essential to improving the education of elementary school mathematics teachers. Pioneer collaborators have found the obstacles to such partnerships to be formidable but surmountable and the fruits sweet.

Puzzles Column

Joe P. Buhler and Elwyn Berlekamp

1. On the parallelogram $ABCD$, point P lies on CD and Q lies on AD. The areas of the triangles ABQ, BCP, and PDQ, are 6, 29, and 17. What's the area of triangle BPQ?

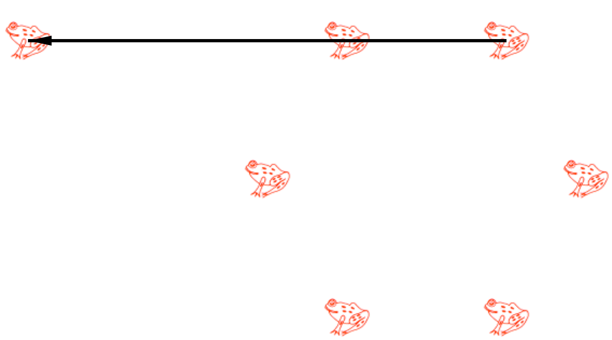


Comment: We heard this one from Rich Schroeppel, who believes that it may have originated with a retired math teacher in Albuquerque.

2. A presliced cake of volume 1 has slices of volume 2^{-k} for various positive integers k. Prove that it is possible to split the cake exactly in half using those pieces, i.e., that there is a collection of pieces whose total volume is $\frac{1}{2}$.

Comment: This appeared on the Problem of the Week at Macalester College, motivated by a lemma in Joel Spencer's paper "Randomization, Derandomization and Antirandomization: Three Games".

3. Frogs start at the vertices of a regular hexagon in the plane. At each second one of the frogs jumps over another, ending up twice as far from the other frog as at the start of the jump.

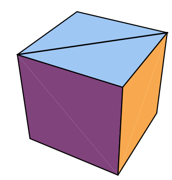


Is it possible for the frogs to choose a sequence of such jumps so that one of them ends up at the exact center of the original regular hexagon?

What if the frogs are allowed to choose between landing twice as far or only half as far?

Comment: This problem appeared on the 2009 Bay Area Mathematical Olympiad for talented high school math students, held late in February.

4. Given eight unit cubes, each having *one* face diagonal drawn on it, can you put them together into the $2 \times 2 \times 2$ cube in such a way that the marked lines form a path from $(0, 0, 0)$ to $(2, 2, 0)$?

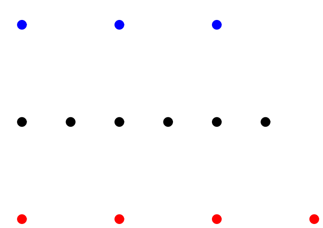


Comment: This puzzle is due to Thomas Colhurst.

5. Find three random variables X, Y, Z, each uniformly distributed on $[0, 1]$, such that their sum is constant. (Since each random variable has expectation $\frac{1}{2}$, the sum must in fact be $\frac{3}{2}$.)

Comment: This problem circulated at the ITA (Information Theory and Applications) conference in San Diego this year. In subsequent discussions we've been surprised at how many different interesting solutions are possible.

6. Given two sets of points on the plane, colored red and blue, a *blocking set* is any set of points that prevents each red point from seeing any blue point (and vice versa). That is, any line segment in the plane with a red and a blue endpoint has a blocking point in its interior. Here is an example, where the blocking set is drawn in black:



For arbitrary positive integers r and b, show how to find r red points and b blue points in the plane admitting an $(r+b-1)$-point blocking set, and so that no three of the red and blue points are collinear. (The example above satisfies the first condition but not the second.)

Comment: We heard this question from Noam Elkies.

March Madness at BAMO

James Sotiros

While the rest of the country was escaping into March Madness watching high school, college and professional basketball players go for the big prize, over 200 budding mathematicians from Bay Area high schools and middle schools walked away with cash and giant trophies!

They were the winners at the BAMO (Bay Area Mathematical Olympiad) Awards Banquet at MSRI on Sunday, March 8, 2009. At this event the highest ranking middle- and high-school students, in grades 6 through 12, were presented with trophies (some of them almost as tall as the recipient) and cash prizes of $50 to $350 in the prestigious Simons Auditorium at MSRI. The BAMO exam and Awards Banquet is sponsored by MSRI Trustee Roger Strauch. The Grand Prize, the Mosse Award, is named for his aunt, Hilde Mosse.

The BAMO exam is demanding and consists of four problems for BAMO8 and five problems for BAMO12, which students have four hours to complete. The test was administered this year on February 24th at schools and community sites throughout the greater San Francisco Bay Area. A record-setting total of 383 students took the BAMO examination this year, including 196 middle school students (grades 6–8) and 187 high school students (grades 9–12).

The size of the "Bay Area" seems to be expanding, as the organizers received requests for participation from Southern California, and the states of Washington and Texas.

The BAMO program was founded in 1998 in conjunction with the math circles program by Paul Zeitz (University of San Francisco), Zvezdelina Stankova (Mills College), and Hugo Rossi (former Deputy Director of MSRI). It is currently directed by Joshua Zucker of MSRI. Emulating famous Eastern European models, the program aims to draw kids to mathematics, to introduce them to the wonders of beautiful mathematical theories, to prepare them for mathematical contests at regional, national, and international levels of competition, and to encourage them to undertake careers linked with mathematics, whether as mathematicians, mathematics educators, economists, or computer scientists.

Kat Wade

From the left: MSRI Trustee Roger Strauch, Grand Prize winners Evan O'Dorney and Julian Hunts, MSRI Director Robert Bryant.

BAMO has been extremely popular since its founding over 20 years ago and has established itself as the most prestigious program in the Bay Area for training in mathematical theory; it is eagerly anticipated each year by students, teachers, and parents. One indication of its success is that three Bay Area students who participated in the BAMO program were selected for the six-member United States team that tied for second place with Russia (after China) among 80 countries at the 2001 International Mathematical Olympiad held in Washington, DC. This early success has been followed by many other accomplishments of students participating in BAMO.

BAMO presents awards for individual participation in different grade categories, and for school (group) participation. Below are the top eighteen individual awards:

Student	**Grade and School**	
BRILLIANCY PRIZE		
Amol Aggarwal	10	Saratoga High School, Saratoga
GRAND PRIZE		
Evan O'Dorney	10	Venture (Home) School, San Ramon
Julian Ziegler Hunts	7	Christa McAuliffe, Saratoga
FIRST PRIZE		
Robert Nishihara	12	Homestead, Cupertino
Amol Aggarwal	10	Saratoga High School, Saratoga
Jeffrey Jiang	8	Miller Middle School, San Jose
Danielle Wang	6	Moreland Middle School, San Jose
SECOND PRIZE		
Taylor Han	11	Henry M. Gunn High, Palo Alto
Albert Gu	10	Saratoga High School, Saratoga
Johnny Ho	8	Miller Middle School, San Jose
THIRD PRIZE		
Mark Holmstrom	8	Britton Middle School, Morgan Hill
Avi Arfin	11	Palo Alto High School
Nathan Pinsker	9	Palo Alto High School
Lynnelle Ye	11	Palo Alto High School
Alan Chang	11	Piedmont Hills High, San Jose
Ashvin Swaminathan	8	The Harker School, San Jose
Kevin Lei	8	Miller Middle School, San Jose
Victor Xu	8	Miller Middle School, San Jose
Nikhil Buduma	8	Miller Middle School, San Jose

The awards were followed by a lunch buffet, and much lively discussion about scores, who won what, and which problem will be "no problem" next year. All left feeling like winners, and we can all be grateful that these students are eagerly beginning preparation for the challenge of taking over the solution of the problems that tomorrow brings.

Forthcoming Workshops

Most of these workshops are offered under the auspices of one of the current programs. For more information about the programs and workshops, see www.msri.org/calendar.

May 11, 2009 to May 13, 2009: *Teaching Undergraduates Mathematics*, organized by William McCallum (The U. of Arizona), Deborah Loewenberg Ball (U. of Michigan), Rikki Blair (Lakeland Comminity College, Ohio), David Bressoud (Macalester College), Amy Cohen-Corwin (Rutgers U.), Don Goldberg (El Camino College), Jim Lewis (U. of Nebraska), Robert Megginson (U. of Michigan), Bob Moses (The Algebra Project), James Donaldson (Howard U.),

June 15, 2009 to July 24, 2009: *MSRI-UP 2009: Coding Theory*, organized by Ivelisse Rubio (U. of Puerto Rico, Humacao), Duane Cooper (Morehouse College), Ricardo Cortez (Tulane U.), Herbert Medina (Loyola Marymount U.), and Suzanne Weekes (Worcester Polytechnic Insitute)

June 15, 2009 to June 26, 2009: *Toric Varieties*, organized by David Cox (Amherst College) and Hal Schenck (U. of Illinois)

June 28, 2009 to July 18, 2009 (at the IAS/Park City Mathematics Institute, Salt Lake City, UT): *IAS/PCMI Summer Program: The Arithmetic of L-functions*, organized by Cristian Popescu (UCSD), Karl Rubin (UC Irvine), Alice Silverberg (UC Irvine).

July 06, 2009 to July 17, 2009: *Random Matrix theory*, organized by Jinho Baik (U. of Michigan), Percy Deift (New York U.), Toufic Suidan (U. of Arizona), Brian Rider (U. of Colorado)

July 06, 2009 to July 24, 2009: *Summer Institute for the Professional Development of Middle School Teachers on Pre-Algebra (Wu Summer Institute)*, organized by Hung-Hsi Wu (U. of California, Berkeley), Stefanie Hassan (Little Lake City School District), Winnie Gilbert (Hacienda La Puente Unified School District), and Sunil Koswatta (Harper College).

July 20, 2009 to July 31, 2009: *Inverse Problems*, organized by Gunther Uhlmann (U. of Washington)

July 20, 2009 to July 24, 2009 (at the University of Utah, Salt Lake City): *Computational Theory of Real Reductive Groups*, organized by Jeffrey Adams (U. of Maryland) , Peter Trapa (U. of Utah), Susana Salamanca (New Mexico State U.), John Stembridge (U. of Michigan), and David Vogan (MIT)

August 03, 2009 to August 14, 2009: *Summer Graduate Workshop: Symplectic and Contact Geometry and Topology*, organized by John Etnyre (Georgia Institute of Technology), Dusa McDuff (Barnard College)

August 14, 2009 to August 15, 2009: *Connections for Women: Symplectic and Contact Geometry and Topology*, organized by Eleny Ionel (Stanford U.), Dusa McDuff (Barnard College, Columbia U.).

August 17, 2009 to August 21, 2009: *Introductory Workshop on Symplectic and Contact Geometry and Topology*, organized by John Etnyre (Georgia Institute of Technology), Dusa McDuff (Barnard College, Columbia U.), and Lisa Traynor (Bryn Mawr)

August 22, 2009 to August 23, 2009: *Connections for Women: Tropical Geometry*, organized by Alicia Dickenstein (U. Buenos Aires), Eva Maria Feichtner (U. Bremen)

August 24, 2009 to August 28, 2009: *Introductory Workshop on Tropical Geometry*, organized by Eva Maria Feichtner (U. Bremen), Ilia Itenberg (U. Strasbourg), chair, Grigory Mikhalkin (U Genève), Bernd Sturmfels (U. C. Berkeley)

September 14, 2009 to September 18, 2009: *Black Holes in Relativity*, organized by Mihalis Dafermos (MIT) and Igor Rodnianski (Princeton)

October 12, 2009 to October 16, 2009: *Tropical Geometry in Combinatorics and Algebra*, organized by Federico Ardila (San Francisco State U.), David Speyer (MIT), chair, Jenia Tevelev (U. Mass Amherst), Lauren Williams (Harvard)

November 16, 2009 to November 20, 2009: *Algebraic Structures in the Theory of Holomorphic Curves*, organized by Mohammed Abouzaid (Clay Mathematics Institute), Yakov Eliashberg (Stanford U.), Kenji Fukaya (Kyoto U.), Eleny Ionel (Stanford U.), Lenny Ng (Duke U.), Paul Seidel (MIT)

November 30, 2009 to December 04, 2009: *Tropical Structures in Geometry and Physics*, organized by Mark Gross (U. of California San Diego), Kentaro Hori (U. of Toronto), Viatcheslav Kharlamov (Université de Strasbourg (Louis Pasteur), Richard Kenyon (Brown U.)

January 21, 2010 to January 22, 2010: *Connections for Women: Homology Theories of Knots and Links*, organized by Elisenda Grigsby (Columbia), Olga Plamenevskaya (SUNY Stony Brook), and Katrin Wehrheim (MIT)

January 25, 2010 to January 29, 2010: *Introductory Workshop: Homology Theories of Knots and Links*, organized by Dylan Thurston (Columbia university)

March 21, 2010 to March 26, 2010: *Symplectic and contact topology and dynamics: puzzles and horizons*, organized by Paul Biran (Tel Aviv U.), John Etnyre (Georgia Institute of Technology), Helmut Hofer (Courant Institute), Dusa McDuff (Barnard College), Leonid Polterovich (Tel Aviv U.),

Current and Recent Workshops

Most recent first. For information see www.msri.org/calendar.

May 04, 2009 to May 06, 2009: *Economic Games and Mechanisms to Address Climate Change*, organized by Rene Carmona (Princeton), Prajit Dutta (Columbia), Chris Jones (U. of North Carolina), Roy Radner (NYU), and David Zetland (UC Berkeley).

April 13, 2009 to April 15, 2009: *Symposium on the Mathematical Dhallenges of Systems Genetics*, organized by Rick Woychick (Director, The Jackson Laboratory) Robert Bryant (Director, MSRI) David Galas (Institute for Systems Biology) Arnold Levine (Institute for Advanced Study) Lee Hood (Institute for Systems Biology) Gary Churchill (The Jackson Laboratory)

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