

Title: Contraction of Areas and Homotopy Types of Mappings

Speaker: Larry Guth

Abstract: The  $k$ -dilation of a mapping measures how much the map stretches  $k$ -dimensional areas. The  $k$ -dilation of a map is at most  $D$  if any  $k$ -dimensional surface in the domain of  $k$ -dimensional volume  $V$  has image with  $k$ -dimensional volume at most  $DV$ . I'm interested in understanding how the  $k$ -dilation constrains the homotopy type of a mapping. We will discuss mappings from the unit  $m$ -sphere to the unit  $n$ -sphere. Perhaps surprisingly, homotopically non-trivial maps can have arbitrarily small  $k$ -dilation for certain  $k$  (depending on  $m$  and  $n$ ).

For example, a homotopically non-trivial map from  $S^7$  to  $S^6$  may have arbitrarily small 5-dilation. But there are also limits to this phenomenon. For example, a homotopically non-trivial map from  $S^7$  to  $S^6$  cannot have arbitrarily small 4-dilation. We discuss some examples and then focus on how to prove lower bounds on the  $k$ -dilation of certain homotopically non-trivial maps.