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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn **Email/Phone:** jhilburn@uoregon.edu

Speaker's Name: Zhiwei Yun

Talk Title: Geometric Representations of Rational Cherednik Algebras

Date: 11 / 17 / 2014 **Time:** 2 : 00 **am** **pm** **(circle one)**

List 6-12 key words for the talk: Rational Cherednik, Algebras, Affine Springer Fiber, Regular Elliptic Numbers

Please summarize the lecture in 5 or fewer sentences:

Yun constructs representations of rational Cherednik algebras using the cohomology of homogeneous affine Springer and Hitchin fibers. He also produced a formula for the dimensions of irreducible representations.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
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- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
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joint w/ Oblomkov

Yun - Geometric Representations of Rational Cherednik Algebras

$\mathfrak{H} \curvearrowright H^*(X)$ want to construct

§ 1 Rational Cherednik Algebra

$W \curvearrowright \mathfrak{t}$ reflection representation of Weyl group

As a vector space

$$\mathfrak{H}^{rat}_{\nu} = \mathrm{Sym}(\mathfrak{t}) \otimes \mathbb{C}[W] \otimes \mathrm{Sym}(\mathfrak{t}^*)$$

~~grading~~ grading: $(-1, \cancel{0})$ $(0, \cancel{0})$ $(1, \cancel{2})$

$\boxed{\text{2nd grading did not make sense}}$

Algebra structure: The 3 tensor factors are subalgebras

$\mathrm{Sym}(\mathfrak{t}) \rtimes W$, $W \ltimes \mathrm{Sym}(\mathfrak{t}^*)$

Most interesting relation is $\xi \in \mathfrak{t}^*$, $x \in \mathfrak{t}$

$$[x, \xi] = \langle \xi, x \rangle - \frac{\nu}{2} \sum_{\alpha \in \Phi} \langle \xi, \alpha^\vee \rangle \langle \alpha, x \rangle r_\alpha$$

central charge

roots for W

reflection with respect to $\alpha = 0$

This is interesting:

finite Hecke alg	affine Hecke alg	double affine Hecke alg
W	$\widetilde{W}$ = affine Weyl group	$\mathfrak{H}$

$\mathfrak{H} \rightsquigarrow \mathfrak{H}^{trig} \rightsquigarrow \mathfrak{H}^{rat}$

E $\quad\quad$ $\mathbb{G}_m$ $\quad\quad$ $\mathbb{G}_a$

Rep Theory of H_ν^{rat}

cat $\mathcal{O}$, verma modules $M_\nu(\tau) = \mathrm{Ind}_{\mathrm{Sym}(\mathfrak{h})\rtimes W}^{H_\nu^{rat}} (\mathrm{triv} \boxtimes \tau)$,

where τ irrep of W

Each verma has a simple quotient

$L_\nu(\tau)$

Basic Question: When is $L_\nu(\tau)$ f.d.

Ex $W = S_2$ $\nu = \frac{d}{2}$ $d > 0$ odd integer

Fact $\mathbb{C}[\xi]/(\xi^d)$ carries an action of H_ν^{rat}

$\mathfrak{h}^* \ni \xi$ acts via mult

$S_2 \ni \sigma$ acts via $\xi \mapsto -\xi$

$\mathfrak{h} \ni x$ acts via $f(\xi) \mapsto f'(\xi) - \frac{d}{2} \frac{f(\xi) - f(-\xi)}{\xi}$.

In fact $L_\nu(\mathrm{triv}) = \mathbb{C}[\xi]/(\xi^d)$

when $\nu > 0$ these are only f.d. irreducible modules.

Varagnolo-Vasserot when is $L_\nu(\mathrm{triv})$ f.d. ?

This happens precisely when $\nu = \frac{d}{m} > 0$ $(d,m) = 1$

where m is a regular elliptic number for W.

Recall $w \in W$ is elliptic if $\mathfrak{h}^w = 0$.

$w \in W$ w regular if it has a regular eigenvector.

$$\{\text{reg elliptic } w \in W\}/\underset{\text{conj}}{\sim} \overset{\Omega}{\hookrightarrow} \mathbb{N}$$

$$w \longmapsto \mathrm{ord}(w)$$

Call a number regular elliptic iff it is in the image of Ω .

Ex $W = S_n \Longrightarrow$ only elliptic # is n

$W = E_8$ 12 values

§2 Affine Springer fibers

G simply connected, (almost) simple gp $/ \mathbb{C}$

Fl = Affine flag variety $= G(\mathbb{C}((\pi)))/I$

~~where~~ $I = ev_0^{-1}(B)$ where $ev_0 : G(\mathbb{C}[[\pi]]) \to G$ and B is a Borel.

$\gamma \in \mathfrak{g}((\pi)) = \mathfrak{g} \otimes \mathbb{C}((\pi))$ gives a vector field on Fl

Fl_γ = zero locus of γ
$= \{ g \in G(\mathbb{C}((\pi)))/I \mid \mathrm{ad}(g^{-1})\gamma \in \mathrm{Lie}\, I \}$

If γ regular s.s. then
Fl_γ = (possibly ∞) union of proj varieties.

Ex $G = SL_2$ $\gamma = \begin{pmatrix} 0 & 1 \\ \pi^3 & 0 \end{pmatrix}$ $Fl_\gamma =$ 

Affine Hecke | Double Affine Hecke
(KL, G-C)

$H_{aff} \circlearrowright K(\text{springer})$ $\quad\quad$ $\mathfrak{h}_\gamma \circlearrowright K(\text{Affine springer})$

Vasserot

$\circlearrowright H(\text{springer})$ $\quad\quad$ $\mathfrak{h}^{trig}_{\nu} \circlearrowright H^*(\text{Affine springer})$

(L) H^{gr}_{aff}

Q: $\mathfrak{h}^{rat}_{\nu} \circlearrowright ?$

§ Homogeneous Affine springer fibers

$\nu = \frac{d}{m}$

Def $\gamma \in \mathfrak{g}((\pi))^{rss}$ is homogeneous of slope ν, if $\forall s \in \mathbb{C}^\times$, $\gamma(s^m \pi) \underset{conj}{\sim} s^d \gamma(\pi)$

Think of γ as a function in π

Ex $\begin{pmatrix} a\pi^3 & \\ & b\pi^3 \end{pmatrix}$ homogeneous of slope 3

$\begin{pmatrix} 0 & 1 \\ \pi^3 & 0 \end{pmatrix}$ ~~homogeneous~~ of slope $\frac{3}{2}$

half sum of positive coroots

$$\mathfrak{g}_\nu = \left\{ \gamma \in \mathfrak{g}((\pi)) \;\middle|\; \begin{array}{c} \gamma(s^m\omega) = Ad(s^{-d\rho^\vee})(s^d \gamma(\omega)) \\ \forall\ s \in \mathbb{C}^\times \end{array} \right\}$$

graded piece in Moy-Prasad filtration on $\mathfrak{g}((\pi))$

Includes all homogeneous elements of slope ν up to conjugation by $G((t))$.

$\mathfrak{g}_\nu \cap \mathfrak{g}((t))^{rss}$ iff m is regular number.

Family of ASF

$$\begin{array}{ccccc} Fl_\gamma & \subseteq & Fl_\nu & \circlearrowleft & G_\nu \\ \downarrow & & \downarrow & & \text{"locally constant"} \\ \gamma & \in & \mathfrak{g}_\nu^{rs} & \circlearrowleft & G_\nu = \text{f.d. reductive group} / \mathbb{C} \end{array}$$

= levi of a parahoric subalgebra $P_\nu \leq G(\mathbb{C}((t)))$

$\pi_1^{G_\nu}(\mathfrak{g}_\nu^{rs}, \gamma) \circlearrowright H^*(Fl_\gamma)$

$\parallel$

$S_\nu \rtimes B_\nu$

S_ν: stabilizer of γ under G_ν

B_ν: braid group

By homogeneity get

$\mathbb{C}^* \circlearrowright Fl_\gamma$

$H^*_{\mathbb{C}^\times}(Fl_\gamma) \longrightarrow H^*_{\varepsilon=1}(Fl_\gamma)$ ← filtration not grading

↑ equivariant parameter

Thm (Oblomkov, Y) $\nu = \frac{d}{m} > 0$, m = regular elliptic number.

1) There is a geometrically defined filtration $P_{\leq i}$ on $H^*_{\varepsilon=1}(Fl_\gamma)^{S_\nu}$, such that

$\mathfrak{h}_\nu^{rat} \circlearrowright Gr^P_* H^*_{\varepsilon=1}(Fl_\gamma)^{S_\nu}$

2) $Gr^P_* H^*_{\varepsilon=1}(Fl_\gamma)^{S_\nu \rtimes B_\nu}$ is

$L_\nu(\mathrm{triv})$.

3) $Gr^P_* H^*_{\varepsilon=1}(Fl_\gamma)^{S_\nu \rtimes B_\nu}$ is a Frobenius algebra under cup product.

4) Dimension formula for $L_\nu(\mathrm{triv})$.

$P_{\leq i}$ comes from deforming affine Springer fiber to smooth variety

$$\begin{tikzcd} Fl_\gamma \arrow[r, hook] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & \text{Hitchin moduli stack over a weighted projective curve} \arrow[d, "f"] \\ \gamma \in A_v \arrow[r] & A \end{tikzcd}$$

$\leftarrow$ general fibers smooth

A: Hitch base

$$\left({}^{p}\tau_{\leq i}\, Rf_*\mathbb{C}\right)_\gamma = P_{\leq i}\, H^*(Fl_\gamma)$$

pf uses Ngo's support theorem