

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn Email/Phone: jhilburn@uoregon.edu

Speaker's Name: Zhiwei Yun

Talk Title: Geometric Representations of Rational Cherednik Algebras

Date: 11 / 17 / 2014 Time: 2 : 00 am **pm** (circle one)

List 6-12 key words for the talk: Rational Cherednik, Algebras, Affine Springer
Fiber, Regular Elliptic Numbers

Please summarize the lecture in 5 or fewer sentences: _____

~~Yun constructs representations of rational Cherednik algebras using the cohomology of homogeneous affine Springer and Hitchin fibers. He also produced a formula for the dimensions of irreducible representations.~~

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
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- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

← sent w/ Oblonkov

Yun - Geometric Representations of Rational Cherednik Algebras

$\mathfrak{h} \subset H^*(X)$ want to construct

§ 1 Rational Cherednik Algebra

$W \curvearrowright \mathfrak{t}$ reflection representations of Weyl group

As a vector space

$$\mathfrak{h}^{\text{rat}} = \text{Sym}(\mathfrak{t}) \otimes \mathbb{C}[W] \otimes \text{Sym}(\mathfrak{t}^*)$$



2nd grading did not make sense

Algebra structure: The 3 tensor factors are subalgebras

$$\text{Sym}(\mathfrak{t}) \rtimes W, \quad W \ltimes \text{Sym}(\mathfrak{t}^*)$$

Most interesting relation is $\{e \in \mathfrak{t}^*, x \in \mathfrak{t}$

$$[x, e] = \langle 1, x \rangle - \frac{1}{2} \sum_{\alpha \in \mathbb{R}} \langle e, \alpha^\vee \rangle \langle x, \alpha \rangle r_\alpha$$

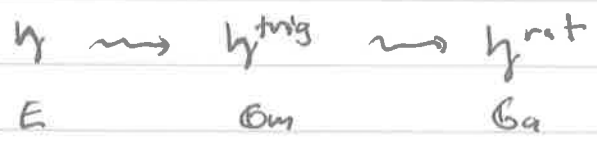
$\alpha \in \mathbb{R}$
 \downarrow
 roots for W

\uparrow
 reflection with respect to $\alpha=0$

\swarrow
 central charge

This is interesting:

finite	affine	double affine
Hecke alg	Hecke alg	Hecke alg
W	\tilde{W} = affine Weyl group	\mathfrak{h}



Rep Theory of $\mathbb{H}_v^{\text{rat}}$

cat \mathcal{O} , Verma modules $M_\nu(z) = \text{Ind}_{\mathfrak{sl}(n) \ltimes W}^{\mathbb{H}_v^{\text{rat}}} (\text{triv} \otimes z)$,
where z irrep of W

Each Verma has a simple quotient
 $L_\nu(z)$

Basic Question: When is $L_\nu(z)$ fid.

Ex $W = S_2$ $\nu = \frac{d}{2}$ $d > 0$ odd integer

Fact $\mathbb{C}\{\xi\}/(\xi^d)$ carries an action of $\mathbb{H}_v^{\text{rat}}$

$\xi \mapsto \xi$ acts via mult

$S_2 \ni \sigma$ acts via $\xi \mapsto -\xi$

$\xi \mapsto x$ acts via $f(\xi) \mapsto f(\xi) - \frac{d}{2} \frac{f(\xi) - f(-\xi)}{\xi}$

In fact $L_\nu(\text{triv}) = \mathbb{C}\{\xi\}/(\xi^d)$

When $\nu > 0$ these are not fid.
irreducible modules.

Verma module - Vassercat When is $L_\nu(\text{triv})$ fid. ?

This happens precisely when $\nu = \frac{d}{m} > 0$ $(d, m) = 1$

where m is a regular elliptic number for W .

Recall $w \in W$ is elliptic if $\epsilon^w = 0$.

$w \in W$ is regular if it has a regular
eigenvector.

$\{ \text{reg elliptic } w \in W \} \xrightarrow[\sim]{\text{ord}} \mathbb{N}$
 $w \longmapsto \text{ord}(w)$

Call a number regular elliptic if it is in the image of Ω

Ex $W = S_n \implies$ only elliptic # is n

$W = E_{\infty}$ 12 values

§2 Affine Springer fibers

G simply connected, (almost) simple gp / \mathbb{C}

$Fl = \text{Affine flag variety} = G(\mathbb{C}((\pi))) / I$

~~the~~ $I = e_{\mathfrak{g}}^{-1}(B)$ where $e_{\mathfrak{g}}: G(\mathbb{C}[[\pi]]) \rightarrow G$
and B is a Borel.

$\gamma \in \mathfrak{g}(\mathbb{C}((\pi))) = \mathfrak{g} \otimes \mathbb{C}((\pi))$ gives a vector field on Fl

$Fl_{\gamma} = \text{zero locus of } \gamma$
 $= \{ g \in G(\mathbb{C}((\pi))) / I \mid \text{ad}(g^{-1})\gamma \in \text{Lie } I \}$

If γ regular s.s. then

$Fl_{\gamma} = (\text{possibly } \infty)$ union of proj varieties.

Ex $G = SL_2$ $\gamma = \begin{pmatrix} 0 & 1 \\ \pi^2 & 0 \end{pmatrix}$ $Fl_{\gamma} = \text{two circles}$

Affine Hecke

Double Affine Hecke

$(Kt, G-C)$

$H_{\text{aff}} \hookrightarrow K(\text{Springer})$

$h_{\nu} \hookrightarrow K(\text{Affine Springer})$

~~H_{aff}~~ $\hookrightarrow H(\text{Springer})$

$v = \text{ker} \pi$

(L) H_{aff}

$h_{\nu}^{\text{rig}} \hookrightarrow H^*(\text{Affine Springer})$

Q: $h_{\nu}^{\text{rig}} \hookrightarrow ?$

\S Homogeneous Affine Springer fibers

$\nu = \frac{d}{m}$

Def $\rightarrow \gamma \in \mathfrak{g}(K[[\pi]])^{\text{reg}}$ is homogeneous of slope ν ,
if $\forall s \in \mathbb{C}^{\times}, \gamma(s^m \pi) \sim_{\text{conj}} s^d \gamma(\pi)$

Think of γ
as a function
in π

Ex $\begin{pmatrix} a\pi^3 & \\ & b\pi^3 \end{pmatrix}$ homogeneous of slope 3

$\begin{pmatrix} 0 & 1 \\ \pi^3 & 0 \end{pmatrix}$ homogeneous of slope $\frac{3}{2}$ half sum of positive roots

$\mathfrak{g}_{\nu} = \left\{ \gamma \in \mathfrak{g}(K[[\pi]]) \mid \gamma(s^m \pi) = \text{Ad}(s^{-d/p}) (s^d \gamma(\pi)) \right\}$
 $\forall s \in \mathbb{C}^{\times}$

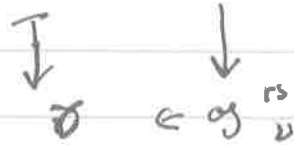
framed piece in Moy-Prasad filtration on $\mathfrak{g}(K[[\pi]])$

Includes all homogeneous elements of slope ν
up to conjugation by $G(K[[\pi]])$.

$\mathfrak{g}_{\nu} \subset \mathfrak{g}(K[[\pi]])^{\text{reg}}$ iff m is regular number.

family of ASF

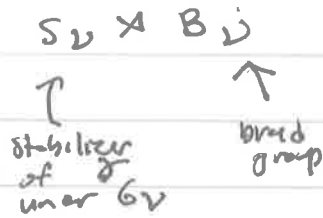
$$FLY \subseteq FLV \hookrightarrow GV$$



« locally constant »

$GV =$ Ad. reductor group / \mathbb{C}
 $=$ Levi of a parabolic subalgebra
 $P_V \subseteq \mathfrak{g}(\mathbb{C})$

$$\pi_1^{GV}(g_v^{rs}, \gamma) \cong H^+(FL\gamma)$$



by homogeneity set

$$\mathbb{C}^* \curvearrowright FL\gamma$$

↳ Action not grading

$$H_{\mathbb{C}^*}^+(FL\gamma) \rightarrow H_{\mathbb{C}^*}^+(FL\gamma)$$

↗ equivalent parameter

Then (Oblonkov, 4) $\nu \geq \frac{d}{m} > 0$, $m =$ regular elliptic number.

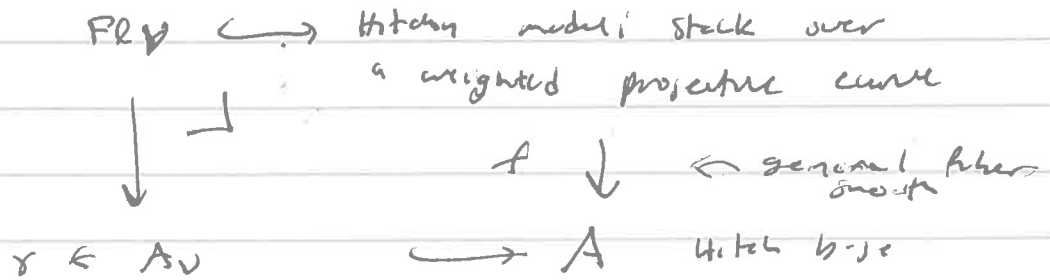
1) There is a geometrically defined filtration $P_{\leq i}$ on $H_{\mathbb{C}^*}^+(FL\gamma)^{S_V}$, such that $L_{\nu}^{int} \subset Gr_{\nu}^P H_{\mathbb{C}^*}^+(FL\gamma)^{S_V}$

2) $Gr_{\nu}^P H_{\mathbb{C}^*}^+(FL\gamma)^{S_V \times B_V}$ is $L_{\nu}(triv)$.

3) $Gr_{\nu}^P H_{\mathbb{C}^*}^+(FL\gamma)^{S_V \times B_V} \supset \mathfrak{F}$ Frobenius algebra under cup product.

4) Dimension formula for $L_{\nu}(triv)$.

$P_{\leq i}$ cones from deformation affine spaces fiber
to smooth varieties



$$(P_{\leq i} Rf_* \mathcal{O})_{\gamma} = P_{\leq i} H^{-1}(FLY_{\gamma})$$

ft uses Ngo's support theorem